Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

FIBER-OPTIC COMMUNICATIONS

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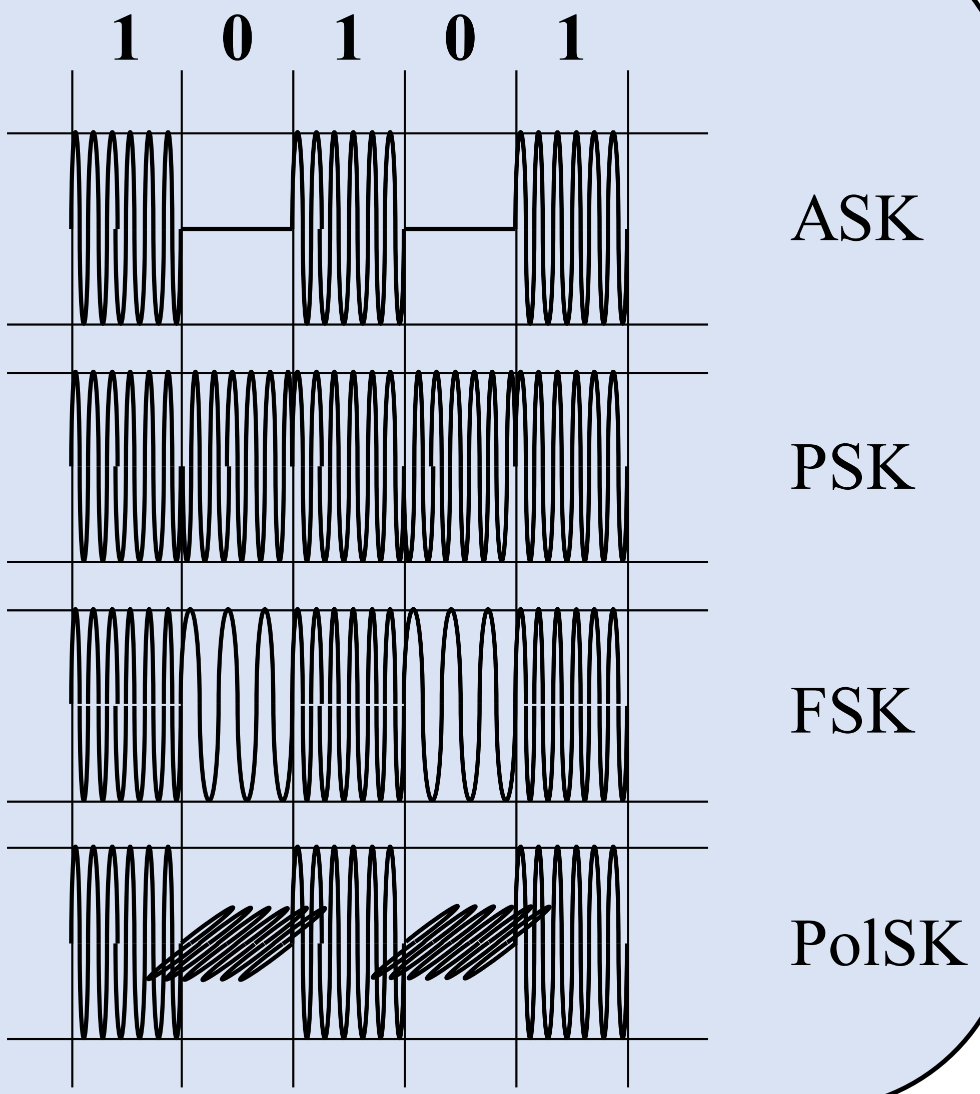
CONTENTS

7. COHERENT DETECTION

- Modulation of Light Properties
- Coherent Receiver
  - Concept
  - Phase Diversity / Polarization Diversity
  - Polarization Multiplexed (PDM) Transmission
- Noise in Coherent Detection
  - Electrical Noise (Thermal)
  - Optical Noise (ASE) – OSNR
  - BER Estimation

- DIGITAL COHERENT DETECTION
  - CHROMATIC DISPERSION COMPENSATION
  - FREQUENCY & PHASE ESTIMATION
  - POLARIZATION TRACKING AND PMD COMPENSATION
- RECENT EXPERIMENTS & COMMERCIAL EQUIPMENT

MODULATION OF LIGHT PROPERTIES



amplitude

phase

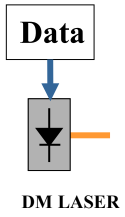
frequency

polarization

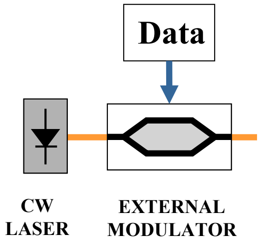
Modulation/ Detection Schemes

Modulation

Direct
- Intensity
- Frequency
- Phase



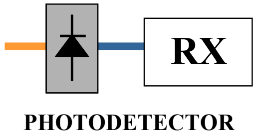
External



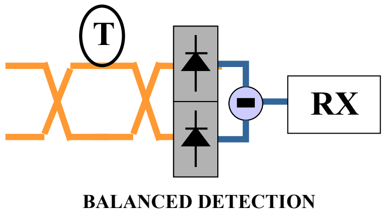
- EAM
  - Intensity
- ERM
  - Phase
  - Polarization
- Mach-Zehnder
  - Intensity
  - Phase

Detection

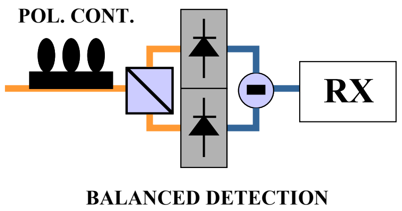
Direct
- Intensity



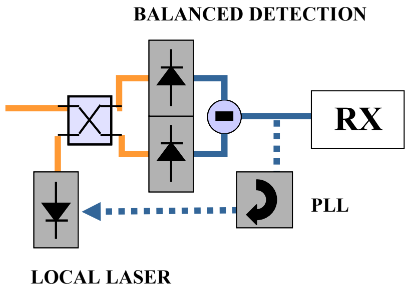
Differential
- Phase
- Frequency
- Polarization



Polarization
- Polarization



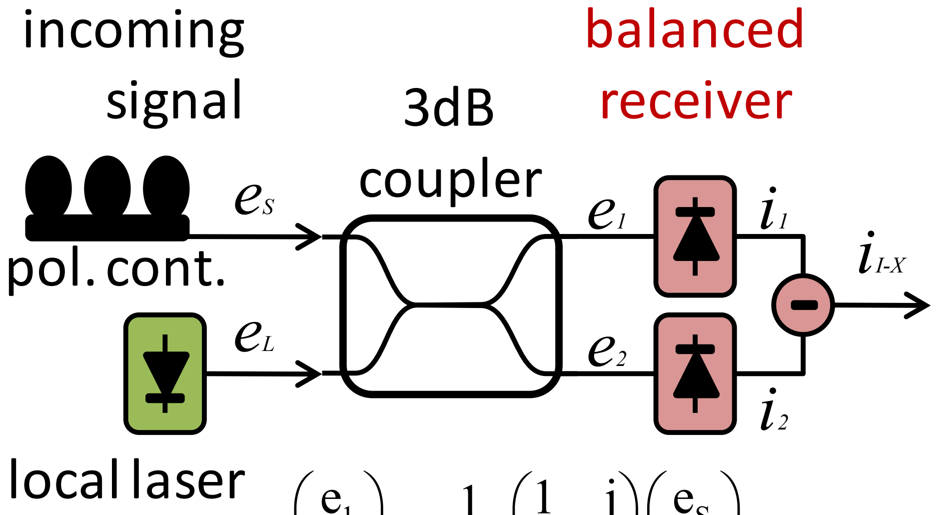
Synchronous
- Intensity
- Phase
- Frequency
- Polarization



COHERENT DETECTION CONCEPT

$$e_S(t) = A_S(t)e^{j\omega_S t} = |A_S(t)|e^{j(\omega_S t+\theta_S(t))}$$

$$e_L(t) = A_L e^{j\omega_L t} = |A_L|e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}$$



$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}}_{C}\begin{pmatrix} e_S \\ e_L \end{pmatrix}$$

Output Signal

information

$$i_{I-X} = 2R|A_S(t)||A_L|\cos(\omega_{FI}t + \theta(t))$$

$$\theta(t) \equiv \theta_S(t) - \theta_L(t)$$

$$\omega_{FI} \equiv \omega_S - \omega_L$$

➡ intermediate frequency ➡

heterodyne detection $\omega_{FI} \neq 0$

homodyne detection $\omega_{FI} = 0$

a phase error of $\pi/2$ kills your signal in homodyne detection

Mathematical Development

$$e_S(t) = A_S(t)e^{j\omega_S t} = |A_S(t)|e^{j(\omega_S t+\theta_S(t))}$$

$$e_L(t) = A_L e^{j\omega_L t} = |A_L|e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}$$

$$C = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & j\\ j & 1\end{pmatrix}$$

$$e_1 = \frac{1}{\sqrt{2}}(e_S + je_L) = \frac{1}{\sqrt{2}}\left(A_S(t)e^{j\omega_S t} + A_L e^{j\left(\omega_L t+\cancel{\frac{\pi}{2}}-\cancel{\frac{\pi}{2}}\right)}\right)$$

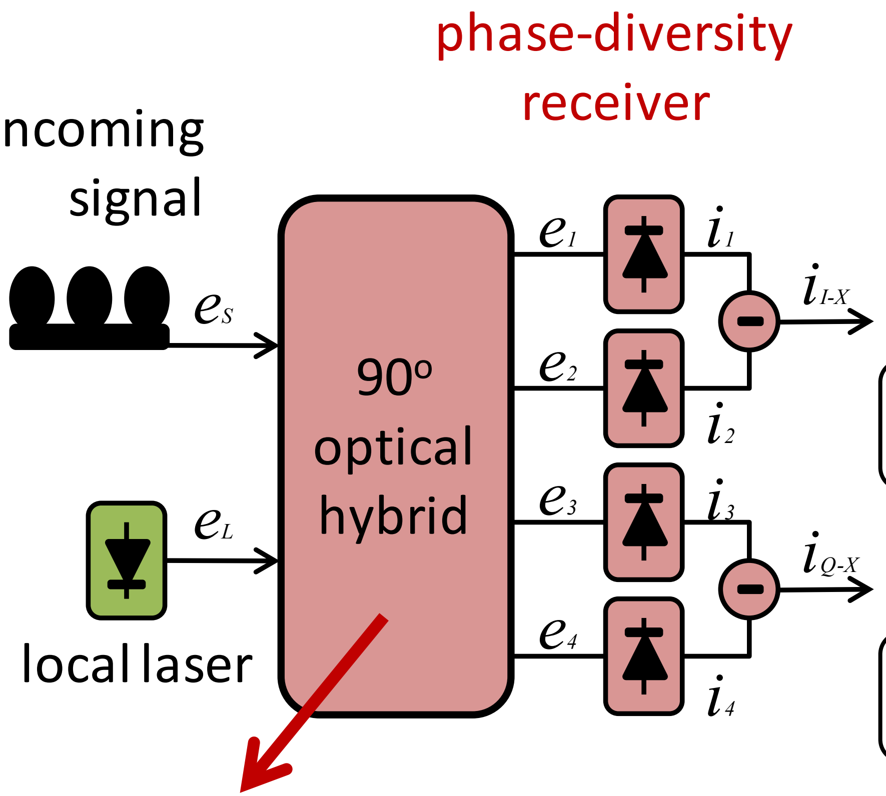
$$e_2 = \frac{1}{\sqrt{2}}(je_S + e_L) = \frac{1}{\sqrt{2}}\left(A_S(t)e^{j\left(\omega_S t+\frac{\pi}{2}\right)} + A_L e^{j\left(\omega_L t-\frac{\pi}{2}\right)}\right)$$

$$i_1 = R|e_1|^2 = R\left[\frac{|A_S(t)|^2}{2} + \frac{|A_L|^2}{2} + |A_S(t)||A_L|\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t+\theta_S(t))}e^{-j(\omega_L t+\theta_L(t))}\right\}}_{\cos((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t))}}_{\cos(\omega_{FI}t+\theta(t))}\right]$$

$$i_2 = R|e_2|^2 = R\left[\frac{|A_S(t)|^2}{2} + \frac{|A_L|^2}{2} + |A_S(t)||A_L|\underbrace{\underbrace{\mathrm{Re}\left\{e^{j\left(\omega_S t+\theta_S(t)+\frac{\pi}{2}\right)}e^{-j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right\}}_{\cos((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t)+\pi)}}_{-\cos(\omega_{FI}t+\theta(t))}\right]$$

$$i_{I-X} = i_1 - i_2 = 2R|A_S(t)||A_L|\cos(\omega_{FI}t+\theta(t))$$

Phase-Diversity Coherent Receiver



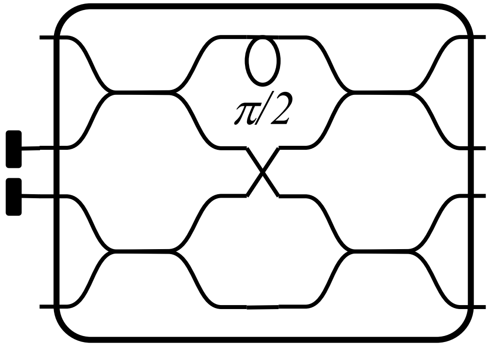
complex information recovery

in-phase signal

$$i_{I-X} = R\left|A_S(t)\right|\left|A_L\right|\cos\left(\omega_{FI}t + \theta(t)\right)$$

in-quadrature signal

$$i_{Q-X} = R\left|A_S(t)\right|\left|A_L\right|\sin\left(\omega_{FI}t + \theta(t)\right)$$



$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} = \frac{1}{2}\underbrace{\begin{pmatrix} j & -1 \\ -1 & j \\ j & j \\ -1 & 1 \end{pmatrix}}_{H}\begin{pmatrix} e_S \\ e_L \end{pmatrix}$$

orthogonal polarization kills your signal in homodyne detection

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Mathematical Development

$$e_S(t) = A_S(t)e^{j\omega_S t} = |A_S(t)|e^{j(\omega_S t + \theta_S(t))}$$

$$e_L(t) = A_L e^{j\omega_L t} = |A_L|e^{j\left(\omega_L t + \theta_L(t) - \frac{\pi}{2}\right)}$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} = \underbrace{\frac{1}{2}\begin{pmatrix} j & -1 \\ -1 & j \\ j & j \\ -1 & 1 \end{pmatrix}}_{H}\begin{pmatrix} e_S \\ e_L \end{pmatrix}$$

$$e_1 = \frac{1}{2}(je_S - e_L) = \frac{1}{2}\left(A_S(t)e^{j\left(\omega_S t + \frac{\pi}{2}\right)} - A_L e^{j\left(\omega_L t - \frac{\pi}{2}\right)}\right) \qquad e_2 = \frac{1}{2}(-e_S + je_L) = \frac{1}{2}\left(-A_S(t)e^{j\omega_S t} + A_L e^{j\omega_L t}\right)$$

$$e_3 = \frac{j}{2}(e_S + e_L) = \frac{j}{2}\left(A_S(t)e^{j\omega_S t} + A_L e^{j\left(\omega_L t - \frac{\pi}{2}\right)}\right) \qquad e_4 = \frac{1}{2}(-e_S + e_L) = \frac{1}{2}\left(-A_S(t)e^{j\omega_S t} + A_L e^{j\left(\omega_L t - \frac{\pi}{2}\right)}\right)$$

$$i_1 = R|e_3|^2 = R\left[\frac{|A_S(t)|^2}{4} + \frac{|A_L|^2}{4} - \frac{|A_S(t)||A_L|}{2}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j\left(\omega_S t + \theta_S(t) + \frac{\pi}{2}\right)}e^{-j\left(\omega_L t + \theta_L(t) - \frac{\pi}{2}\right)}\right\}}_{\cos((\omega_S - \omega_L)t + \theta_S(t) - \theta_L(t) + \pi)}}_{-\cos(\omega_{FI} t + \theta(t))}\right]$$

$$i_2 = R|e_4|^2 = R\left[\frac{|A_S(t)|^2}{4} + \frac{|A_L|^2}{4} - \frac{|A_S(t)||A_L|}{2}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t + \theta_S(t))}e^{-j(\omega_L t + \theta_L(t))}\right\}}_{\cos((\omega_S - \omega_L)t + \theta_S(t) - \theta_L(t))}}_{\cos(\omega_{FI} t + \theta(t))}\right]$$

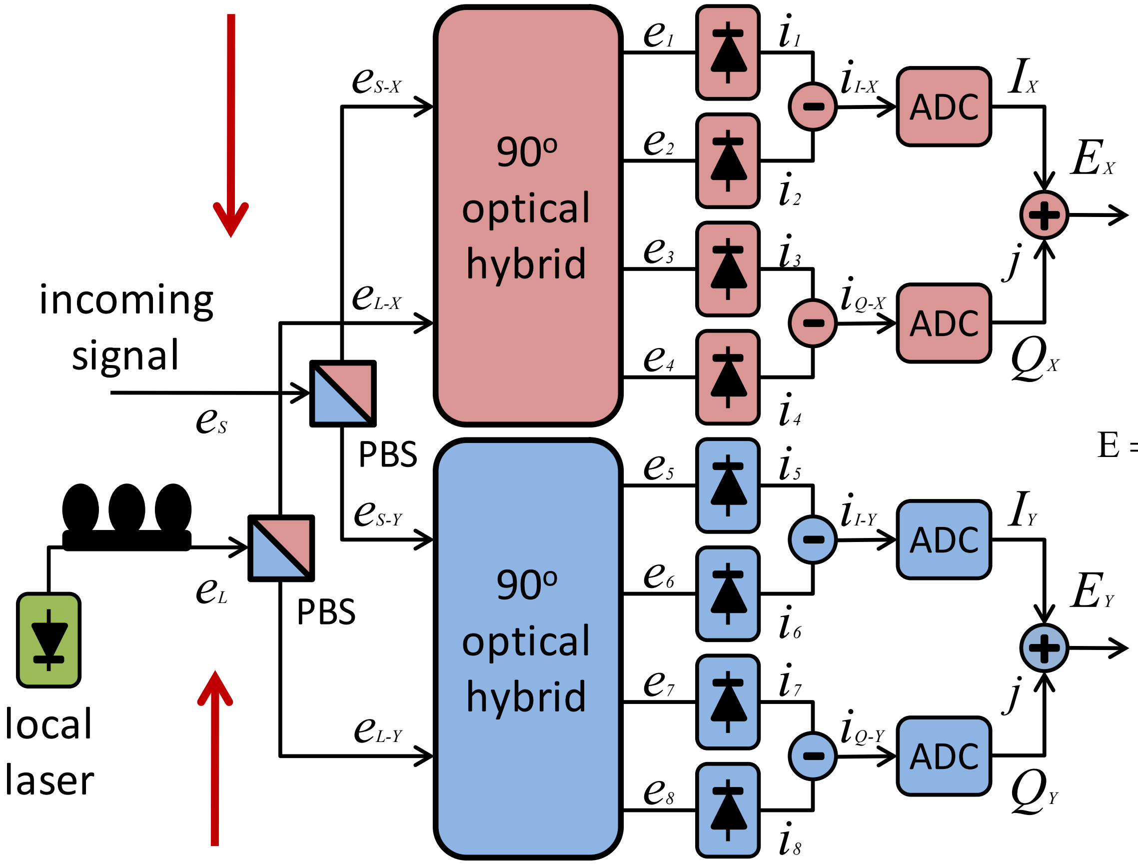
$$i_3 = R|e_3|^2 = R\left[\frac{|A_S(t)|^2}{4} + \frac{|A_L|^2}{4} + \frac{|A_S(t)||A_L|}{2}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t + \theta_S(t))}e^{-j\left(\omega_L t + \theta_L(t) - \frac{\pi}{2}\right)}\right\}}_{\cos\left((\omega_S - \omega_L)t + \theta_S(t) - \theta_L(t) + \frac{\pi}{2}\right)}}_{\sin(\omega_{FI} t + \theta(t))}\right]$$

$$i_4 = R|e_4|^2 = R\left[\frac{|A_S(t)|^2}{4} + \frac{|A_L|^2}{4} - \frac{|A_S(t)||A_L|}{2}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t + \theta_S(t))}e^{-j\left(\omega_L t + \theta_L(t) - \frac{\pi}{2}\right)}\right\}}_{\cos\left((\omega_S - \omega_L)t + \theta_S(t) - \theta_L(t) + \frac{\pi}{2}\right)}}_{\sin(\omega_{FI} t + \theta(t))}\right]$$

$$i_{I-X} = i_1 - i_2 = R|A_S(t)||A_L|\cos(\omega_{FI} t + \theta(t)) \qquad i_{Q-X} = i_3 - i_4 = R|A_S(t)||A_L|\sin(\omega_{FI} t + \theta(t))$$

Phase & Polarization-Diversity

$$e_S(t)=\begin{pmatrix} e_{S-X}(t) \\ e_{S-Y}(t) \end{pmatrix}=|A_S(t)|\begin{pmatrix} \sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} e^{j(\omega_S t+\theta_S(t))}$$



$$e_L(t)=\begin{pmatrix} e_{L-X}(t) \\ e_{L-Y}(t) \end{pmatrix}=\frac{|A_L|}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}$$

full information recovery

$$E_X=R\sqrt{\frac{\alpha}{2}}|A_S(t)||A_L|e^{j(\omega_{FI}t+\theta(t)+\Delta\theta(t))}$$

X-polarization signal

$$E=\frac{R}{\sqrt{2}}|A_S(t)||A_L|\begin{pmatrix} \sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} e^{j\left(\omega_{FI}t+\overbrace{\theta_S(t)-\theta_L(t)}^{\theta(t)}\right)}$$

Y-polarization signal

$$E_Y=R\sqrt{\frac{1-\alpha}{2}}|A_S(t)||A_L|e^{j(\omega_{FI}t+\theta(t))}$$

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Mathematical Development (I)

$$e_S(t)=\begin{pmatrix}e_{S-X}(t)\\ e_{S-Y}(t)\end{pmatrix}=\begin{pmatrix}A_{S-X}(t)\\ A_{S-Y}(t)\end{pmatrix}e^{j\omega_S t}=|A_S(t)|\begin{pmatrix}\sqrt{\alpha}\cdot e^{j\Delta\theta(t)}\\ \sqrt{1-\alpha}\end{pmatrix}e^{j(\omega_S t+\theta_S(t))}$$

$$e_L(t)=\begin{pmatrix}e_{L-X}(t)\\ e_{L-Y}(t)\end{pmatrix}=\begin{pmatrix}A_{L-X}(t)\\ A_{L-Y}(t)\end{pmatrix}e^{j\omega_S t}=\frac{|A_L|}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}$$

$$\begin{pmatrix}e_1 & e_5\\ e_2 & e_6\\ e_3 & e_7\\ e_4 & e_8\end{pmatrix}=\underbrace{\frac{1}{2}\begin{pmatrix}j & -1\\ -1 & j\\ j & j\\ -1 & 1\end{pmatrix}}_{H}\begin{pmatrix}e_S^T\\ e_L^T\end{pmatrix}$$

$$e_1=\frac{1}{2}(je_{S-X}-e_{L-X})=\frac{1}{2}\left(\sqrt{\alpha}|A_S(t)|e^{j\left(\omega_S t+\theta_S(t)+\Delta\theta(t)+\frac{\pi}{2}\right)}-\frac{|A_L|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$e_2=\frac{1}{2}(-e_{S-X}+je_{L-X})=\frac{1}{2}\left(-\sqrt{\alpha}|A_S(t)|e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}+\frac{|A_L|}{\sqrt{2}}e^{j(\omega_L t+\theta_L(t))}\right)$$

$$e_3=\frac{j}{2}(e_{S-X}+e_{L-X})=\frac{j}{2}\left(\sqrt{\alpha}|A_S(t)|e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}+\frac{|A_L|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$e_4=\frac{1}{2}(-e_{S-X}+e_{L-X})=\frac{1}{2}\left(-\sqrt{\alpha}|A_S(t)|e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}+\frac{|A_L|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$e_5=\frac{1}{2}(je_{S-Y}-e_{L-Y})=\frac{1}{2}\left(\sqrt{1-\alpha}|A_S(t)|e^{j\left(\omega_S t+\theta_S(t)+\frac{\pi}{2}\right)}-\frac{|A_L|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$e_6=\frac{1}{2}(-e_{S-Y}+je_{L-Y})=\frac{1}{2}\left(-\sqrt{1-\alpha}|A_S(t)|e^{j(\omega_S t+\theta_S(t))}+\frac{|A_L|}{\sqrt{2}}e^{j(\omega_L t+\theta_L(t))}\right)$$

Mathematical Development (II)

$$e_7=\frac{j}{2}\left(e_{S-Y}+e_{L-Y}\right)=\frac{j}{2}\left(\sqrt{1-\alpha}\,\left|A_S(t)\right|e^{j(\omega_S t+\theta_S(t))}+\frac{\left|A_L\right|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$e_8=\frac{1}{2}\left(-e_{S-Y}+e_{L-Y}\right)=\frac{1}{2}\left(-\sqrt{1-\alpha}\,\left|A_S(t)\right|e^{j(\omega_S t+\theta_S(t))}+\frac{\left|A_L\right|}{\sqrt{2}}e^{j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right)$$

$$i_1=R\left|e_1\right|^2=R\left[\alpha\frac{\left|A_S(t)\right|^2}{4}+\frac{\left|A_L\right|^2}{8}-\sqrt{\alpha}\frac{\left|A_S(t)\right|\left|A_L\right|}{2\sqrt{2}}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j\left(\omega_S t+\theta_S(t)+\Delta\theta(t)+\frac{\pi}{2}\right)}e^{-j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right\}}_{\cos\left((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t)+\Delta\theta(t)+\pi\right)}}_{-\cos\left(\omega_{FI}t+\theta(t)+\Delta\theta(t)\right)}\right]$$

$$i_2=R\left|e_2\right|^2=R\left[\alpha\frac{\left|A_S(t)\right|^2}{4}+\frac{\left|A_L\right|^2}{8}-\sqrt{\alpha}\frac{\left|A_S(t)\right|\left|A_L\right|}{2\sqrt{2}}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}e^{-j(\omega_L t+\theta_L(t))}\right\}}_{\cos\left((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t)+\Delta\theta(t)\right)}}_{\cos\left(\omega_{FI}t+\theta(t)+\Delta\theta(t)\right)}\right]$$

$$i_3=R\left|e_3\right|^2=R\left[\alpha\frac{\left|A_S(t)\right|^2}{4}+\frac{\left|A_L\right|^2}{8}+\sqrt{\alpha}\frac{\left|A_S(t)\right|\left|A_L\right|}{2\sqrt{2}}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}e^{-j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right\}}_{\cos\left((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t)+\Delta\theta(t)+\frac{\pi}{2}\right)}}_{\sin\left(\omega_{FI}t+\theta(t)+\Delta\theta(t)\right)}\right]$$

$$i_4=R\left|e_4\right|^2=R\left[\alpha\frac{\left|A_S(t)\right|^2}{4}+\frac{\left|A_L\right|^2}{8}-\sqrt{\alpha}\frac{\left|A_S(t)\right|\left|A_L\right|}{2\sqrt{2}}\underbrace{\underbrace{\mathrm{Re}\left\{e^{j(\omega_S t+\theta_S(t)+\Delta\theta(t))}e^{-j\left(\omega_L t+\theta_L(t)-\frac{\pi}{2}\right)}\right\}}_{\cos\left((\omega_S-\omega_L)t+\theta_S(t)-\theta_L(t)+\Delta\theta(t)+\frac{\pi}{2}\right)}}_{\sin\left(\omega_{FI}t+\theta(t)+\Delta\theta(t)\right)}\right]$$

Mathematical Development (III)

$$i_{I-X} = i_1 - i_2 = R\sqrt{\frac{\alpha}{2}}\left|A_S(t)\right|\left|A_L\right|\cos\left(\omega_{FI}t + \theta(t) + \Delta\theta(t)\right)$$

$$i_{Q-X} = i_3 - i_4 = R\sqrt{\frac{\alpha}{2}}\left|A_S(t)\right|\left|A_L\right|\sin\left(\omega_{FI}t + \theta(t) + \Delta\theta(t)\right)$$

$$i_{I-Y} = i_5 - i_6 = R\sqrt{\frac{1-\alpha}{2}}\left|A_S(t)\right|\left|A_L\right|\cos\left(\omega_{FI}t + \theta(t)\right)$$

$$i_{Q-Y} = i_7 - i_8 = R\sqrt{\frac{1-\alpha}{2}}\left|A_S(t)\right|\left|A_L\right|\sin\left(\omega_{FI}t + \theta(t)\right)$$

$$i_X = i_{I-X} + j\cdot i_{Q-X} = R\sqrt{\frac{\alpha}{2}}\left|A_S(t)\right|\left|A_L\right| e^{j\left(\omega_{FI}t + \theta(t) + \Delta\theta(t)\right)}$$

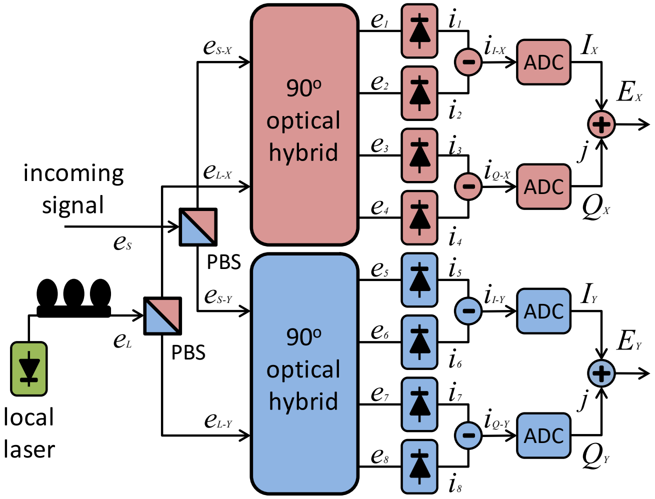
$$i_Y = i_{I-Y} + j\cdot i_{Q-Y} = R\sqrt{\frac{1-\alpha}{2}}\left|A_S(t)\right|\left|A_L\right| e^{j\left(\omega_{FI}t + \theta(t)\right)}$$

$$E = \frac{R}{\sqrt{2}}\left|A_S(t)\right|\left|A_L\right|\begin{pmatrix}\sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha}\end{pmatrix} e^{j\left(\omega_{FI}t + \overbrace{\theta_S(t) - \theta_L(t)}^{\theta(t)}\right)}$$

Polarization Division Multiplexing (PDM)

$$e_S(t) = \underbrace{e_S^{\parallel}(t)}_{\text{Channel } ||} + \underbrace{e_S^{\perp}(t)}_{\text{Channel } \perp} \rightarrow \begin{cases} e_S^{\parallel}(t) = \left|A_S^{\parallel}(t)\right| \begin{pmatrix} \sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} e^{j\left(\omega_S t + \theta_S^{\parallel}(t)\right)} \\ e_S^{\perp}(t) = \left|A_S^{\perp}(t)\right| \begin{pmatrix} \sqrt{1-\alpha} \\ -\sqrt{\alpha}\cdot e^{-j\Delta\theta(t)} \end{pmatrix} e^{j\left(\omega_S t + \theta_S^{\perp}(t)\right)} \end{cases}$$

orthogonal components



$$e_L(t) = \begin{pmatrix} e_{L-X}(t) \\ e_{L-Y}(t) \end{pmatrix} = \frac{|A_L|}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{j\left(\omega_L t + \theta_L(t) - \frac{\pi}{2}\right)}$$

$$E = E_{\parallel} + E_{\perp}$$

$$\begin{cases} E_{\parallel} = \dfrac{R}{\sqrt{2}} \left|A_S^{\parallel}(t)\right| \left|A_L\right| \begin{pmatrix} \sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\parallel}(t) - \theta_L(t)}^{\theta_{\parallel}(t)}\right)} \\ E_{\perp} = \dfrac{R}{\sqrt{2}} \left|A_S^{\perp}(t)\right| \left|A_L\right| \begin{pmatrix} \sqrt{1-\alpha} \\ -\sqrt{\alpha}\cdot e^{-j\Delta\theta(t)} \end{pmatrix} e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\perp}(t) - \theta_L(t)}^{\theta_{\perp}(t)}\right)} \end{cases}$$

mixed information

Polarization Division Multiplexing (PDM)

$$E_{\parallel} = \frac{R}{\sqrt{2}} \left|A_S^{\parallel}(t)\right| \left|A_L\right| \begin{pmatrix} \sqrt{\alpha} \cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\parallel}(t) - \theta_L(t)}^{\theta_{\parallel}(t)}\right)}$$

$$E_{\perp} = \frac{R}{\sqrt{2}} \left|A_S^{\perp}(t)\right| \left|A_L\right| \begin{pmatrix} \sqrt{1-\alpha} \\ -\sqrt{\alpha} \cdot e^{-j\Delta\theta(t)} \end{pmatrix} e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\perp}(t) - \theta_L(t)}^{\theta_{\perp}(t)}\right)}$$

unitary channel (no PDL*)

$E_{X\text{-}in}$ → **Inverse Channel Matrix $C^{\dagger}$** → $E_{X\text{-}out}$

$E_{Y\text{-}in}$ → **Inverse Channel Matrix $C^{\dagger}$** → $E_{Y\text{-}out}$

$$\begin{pmatrix} \sqrt{\alpha} \cdot e^{-j\Delta\theta(t)} & \sqrt{1-\alpha} \\ \sqrt{1-\alpha} & -\sqrt{\alpha} \cdot e^{j\Delta\theta(t)} \end{pmatrix}$$

$$C^{\dagger}\left(E_{\parallel} + E_{\perp}\right) \to A_{\parallel} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_{\perp} \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{\alpha} \cdot e^{-j\Delta\theta(t)} & \sqrt{1-\alpha} \\ \sqrt{1-\alpha} & -\sqrt{\alpha} \cdot e^{j\Delta\theta(t)} \end{pmatrix} \begin{pmatrix} \sqrt{\alpha} \cdot e^{j\Delta\theta(t)} \\ \sqrt{1-\alpha} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{\alpha} \cdot e^{-j\Delta\theta(t)} & \sqrt{1-\alpha} \\ \sqrt{1-\alpha} & -\sqrt{\alpha} \cdot e^{j\Delta\theta(t)} \end{pmatrix} \begin{pmatrix} \sqrt{1-\alpha} \\ -\sqrt{\alpha} \cdot e^{-j\Delta\theta(t)} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$E_{X-out} = \frac{R}{\sqrt{2}} \left|A_S^{\parallel}(t)\right| \left|A_L\right| e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\parallel}(t) - \theta_L(t)}^{\theta_{\parallel}(t)}\right)}$$

$$E_{Y-out} = \frac{R}{\sqrt{2}} \left|A_S^{\perp}(t)\right| \left|A_L\right| e^{j\left(\omega_{FI} t + \overbrace{\theta_S^{\perp}(t) - \theta_L(t)}^{\theta_{\perp}(t)}\right)}$$

recovered information

PDL: Polarization-Dependent Loss

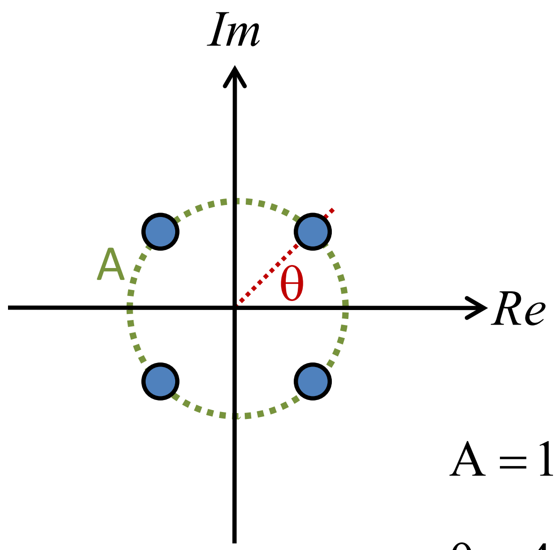
Polarization Division Multiplexing (PDM)

$$E_{X-out} = \frac{R}{\sqrt{2}}\left|A_S^{\parallel}(t)\right|\left|A_L\right| e^{j\left(\omega_{FI}t + \overbrace{\theta_S^{\parallel}(t) - \theta_L(t)}^{\theta_{\parallel}(t)}\right)}$$

$$E_{Y-out} = \frac{R}{\sqrt{2}}\left|A_S^{\perp}(t)\right|\left|A_L\right| e^{j\left(\omega_{FI}t + \overbrace{\theta_S^{\perp}(t) - \theta_L(t)}^{\theta_{\perp}(t)}\right)}$$

Modulation Examples

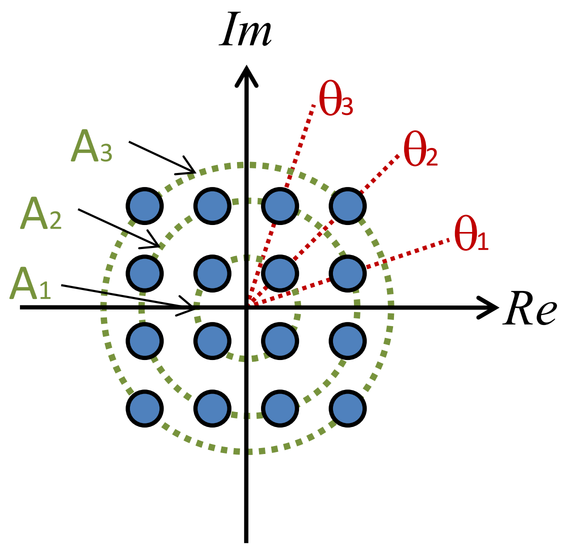
Normalized Power: $P = A^2$



$A = 1$

$\theta = 45^o$

QPSK



16-QAM

$$A_1 = \frac{A}{\sqrt{5}} \approx 0.45 \cdot A$$

$$A_2 = A$$

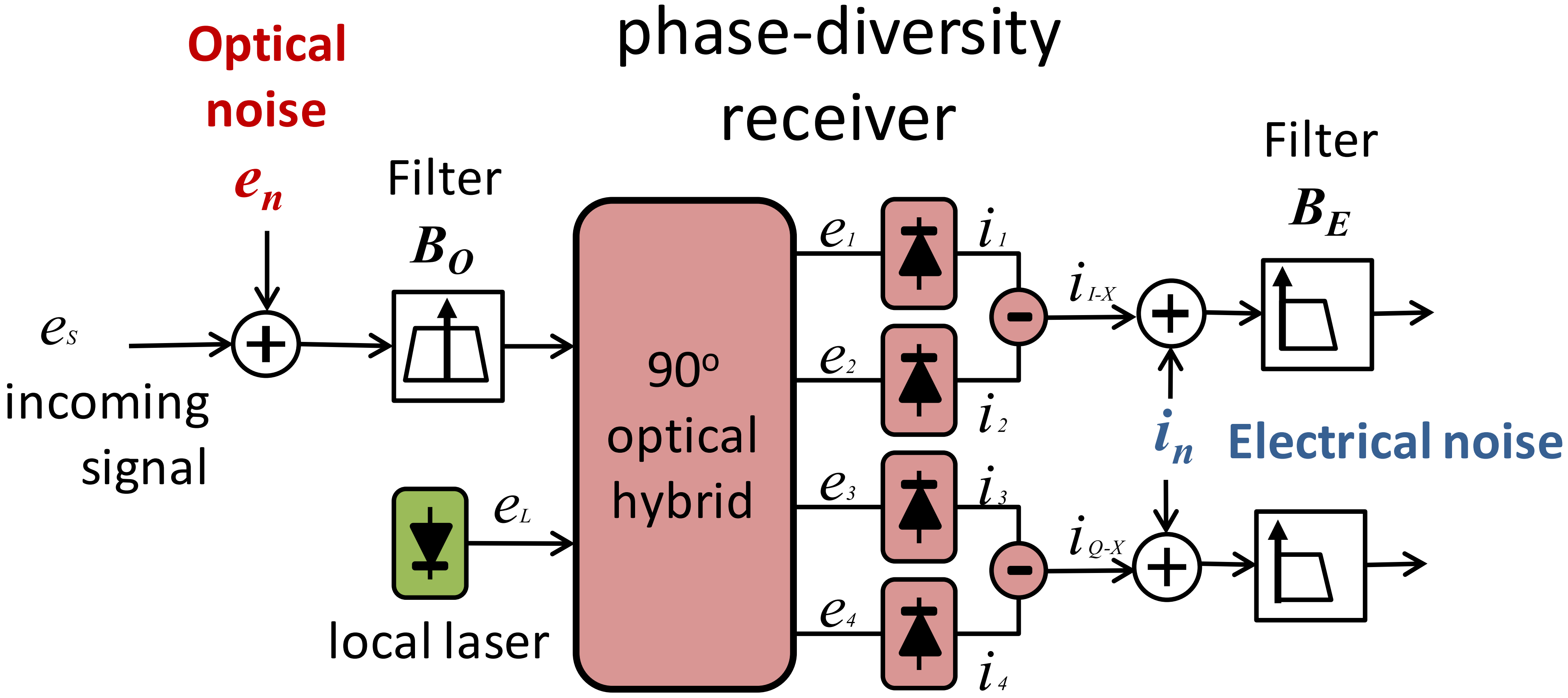
$$A_3 = \frac{3A}{\sqrt{5}} \approx 1.34 \cdot A$$

$$\theta_1 \approx 18^o$$

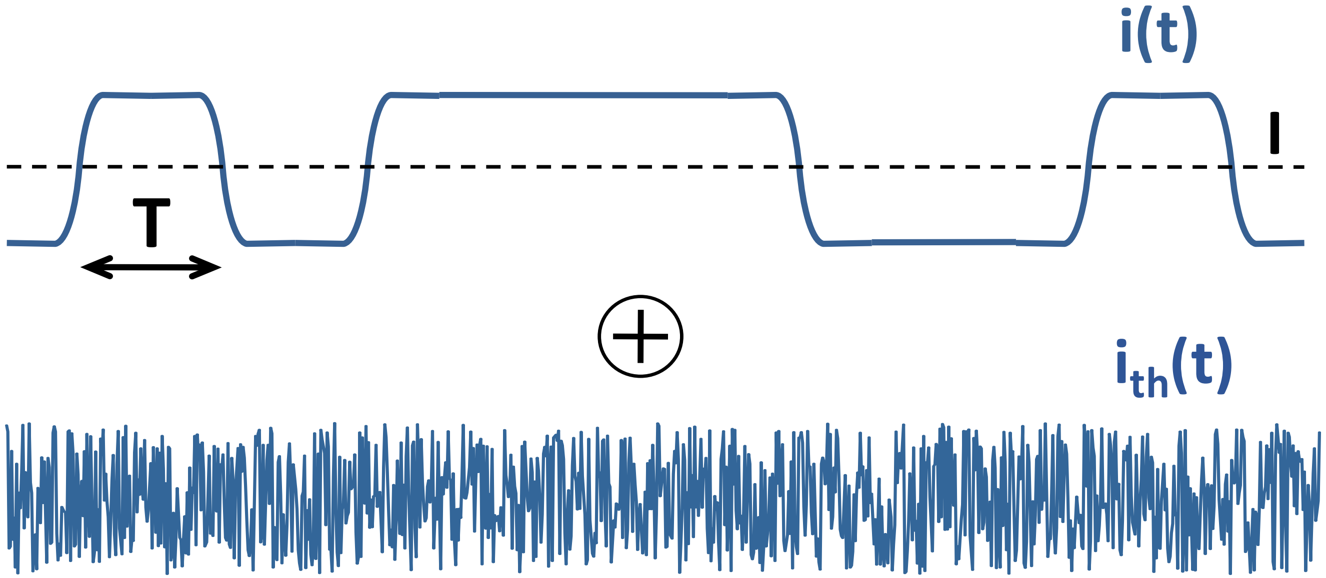
$$\theta_2 = 45^o$$

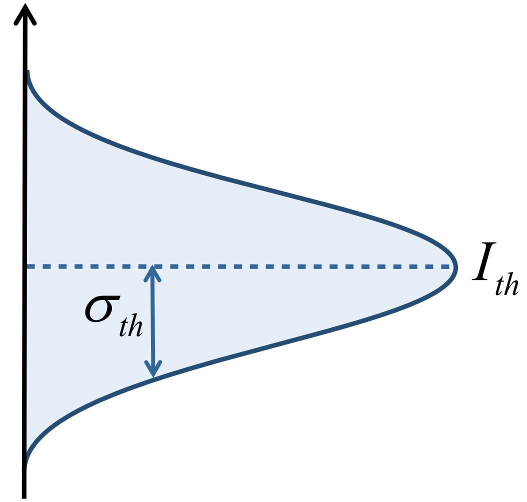
$$\theta_3 \approx 72^o$$

NOISE IN COHERENT DETECTION



Electrical Noise: Thermal





AWGN Process

$$I_{th} = 0 \quad [A]$$

$$\sigma_{th}^2 = 4\underbrace{\frac{K_B T}{R_L}}_{S_{th}} B_e \quad \left[A^2\right]$$

K_B: Boltzmann's Constant $[J/K]$

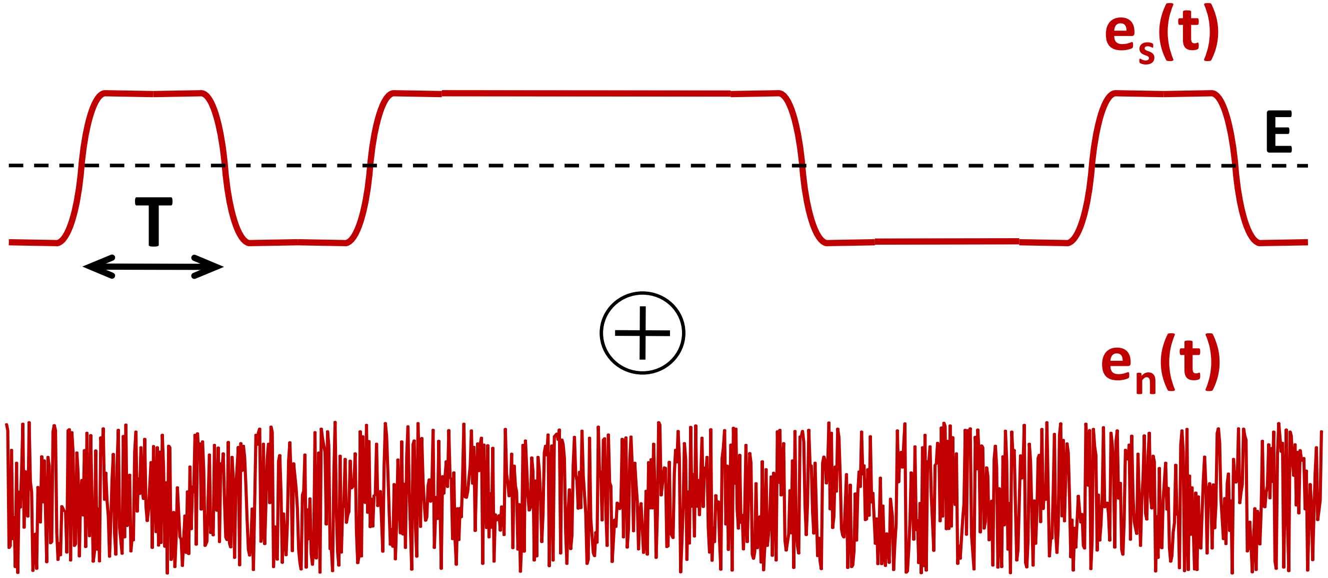
T: Temperature $[K]$

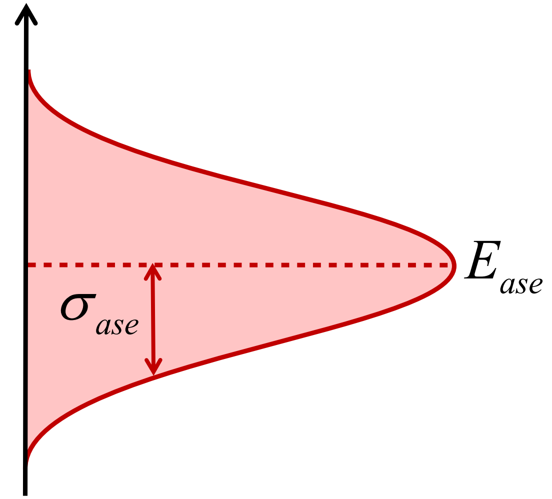
R_L: Circuit Load $[\Omega]$

B_e: Noise Equivalent Bandwidth $[Hz]$

(Typical Value) $2{\cdot}10^{-22}$ ← S_{th}: Noise Spectral Density $[A^2/Hz]$

Optical Noise: ASE





AWGN Process

$$E_{ase} = 0 \quad [W^{1/2}]$$

$$\underbrace{\sigma_{ase}^2}_{P_{ase}} \approx S_{ase}(f_c) B_o \quad [W]$$

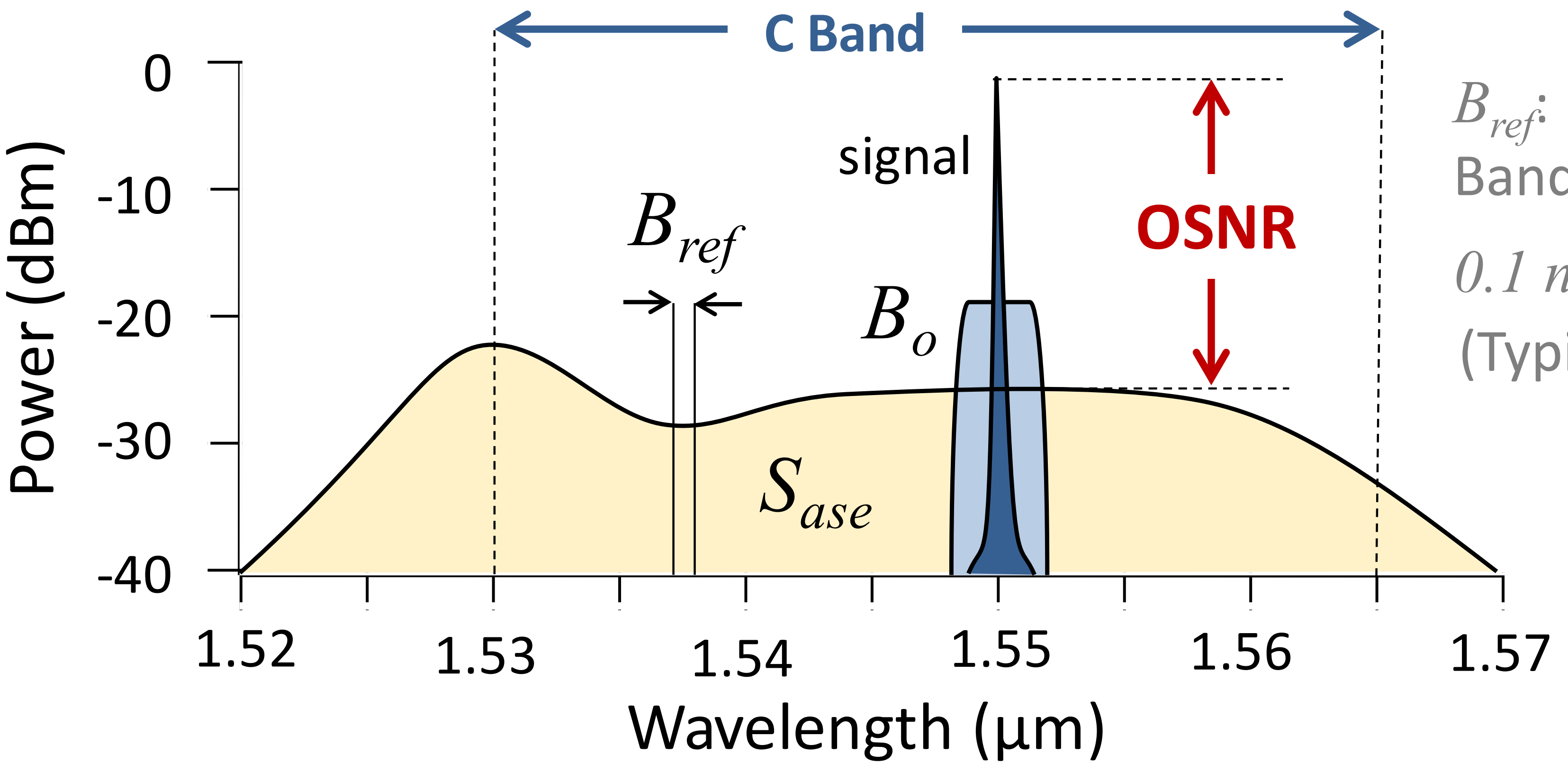
ASE: Amplified Spontaneous Emission

S_{ase}: ASE Spectral Density $[W/Hz]$

B_o: Optical Bandwidth $[Hz]$

Optical Signal to Noise Ratio (OSNR)

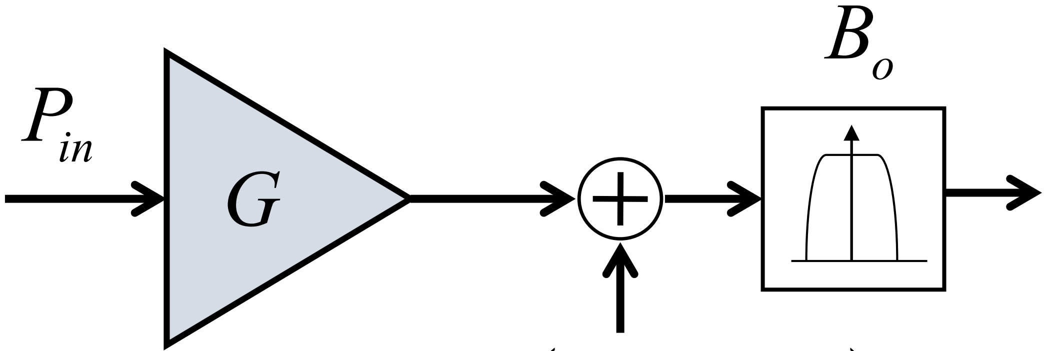
$$OSNR_{B_{ref}} = \frac{P_s}{S_{ase}(f_c)B_{ref}} \rightarrow P_{ase} = S_{ase}(f_c)B_o = \frac{P_s}{OSNR_{B_{ref}}}\frac{B_o}{B_{ref}}$$



B_{ref}: Reference Bandwidth [Hz]

0.1 nm (12.5 GHz) (Typical Value)

ASE in Erbium-Doped Fiber Amps. (EDFA)



$$P_{out} = G(f_c)P_{in} \qquad \text{(Signal)}$$

$$P_{ase} \approx S_{ase}(f_c)B_o \qquad \text{(Noise)}$$

$$S_{ase} = hf\left(G(f)-1\right)2n_{sp}$$

h: Planck's Constant $[J{\cdot}s]$

G: Amplifier's Gain

$n_{sp} \geq 1$: Spontaneous Em. Factor

QL: Quantum Limit

$$OSNR_{out} = \frac{\overbrace{GP_{in}}^{P_{out}}}{\underbrace{hf_c(G-1)2n_{sp}B_{ref}}_{P_{ase}}} \approx \frac{P_{in}}{hf_c 2n_{sp}B_{ref}}$$

Noise Figure

$$F \equiv \frac{OSNR_{in}}{OSNR_{out}} \approx \frac{\overbrace{P_{in}/hf_c \cdot B_{ref}}^{OSNR_{QL}}}{P_{in}/hf_c 2n_{sp}B_{ref}} = \underbrace{2n_{sp}}_{\geq 2}$$

Noise Figure Measurement

$$F_{dB} = P_{in}^{dBm} - OSNR_{out}^{dB} - \underbrace{10\log_{10}\left(hf_c B_{ref}\right)}_{\substack{Quantum\ \ Noise \\ (-58dBm\ @\ 0.1nm)}}$$

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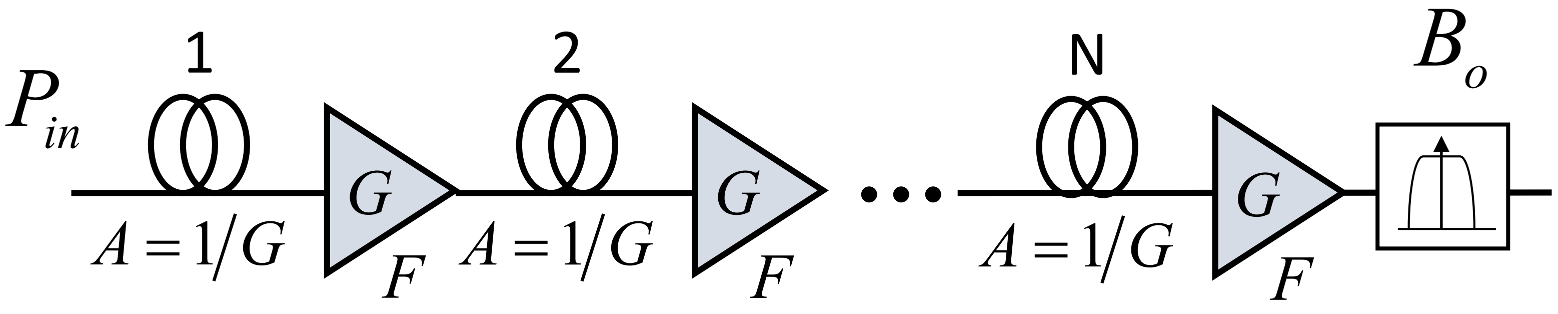
OSNR in an Link with EDFAs

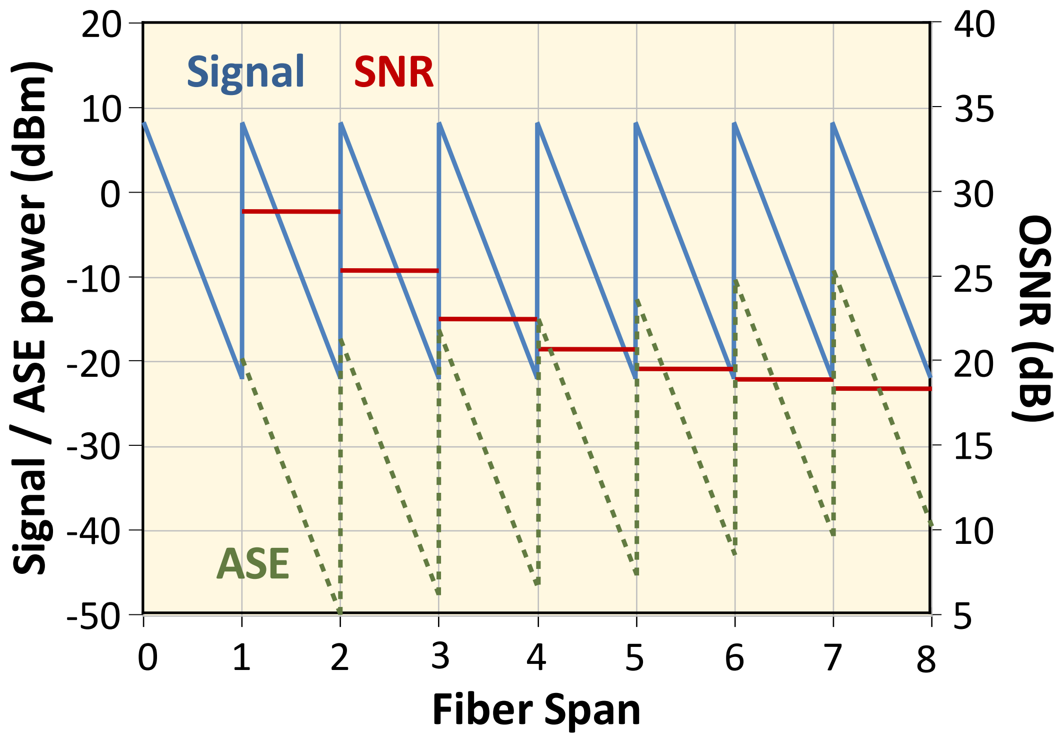


$$P_{out} = P_{in}$$

$$P_{ase}^{tot} = NP_{ase}$$

$$OSNR_{link} = \frac{P_{in}}{NP_{ase}}$$

$$P_{ase} \approx hf_c (G-1) F \cdot B_0$$



BIT ERROR RATIO (BER) ESTIMATION

Monte Carlo Method

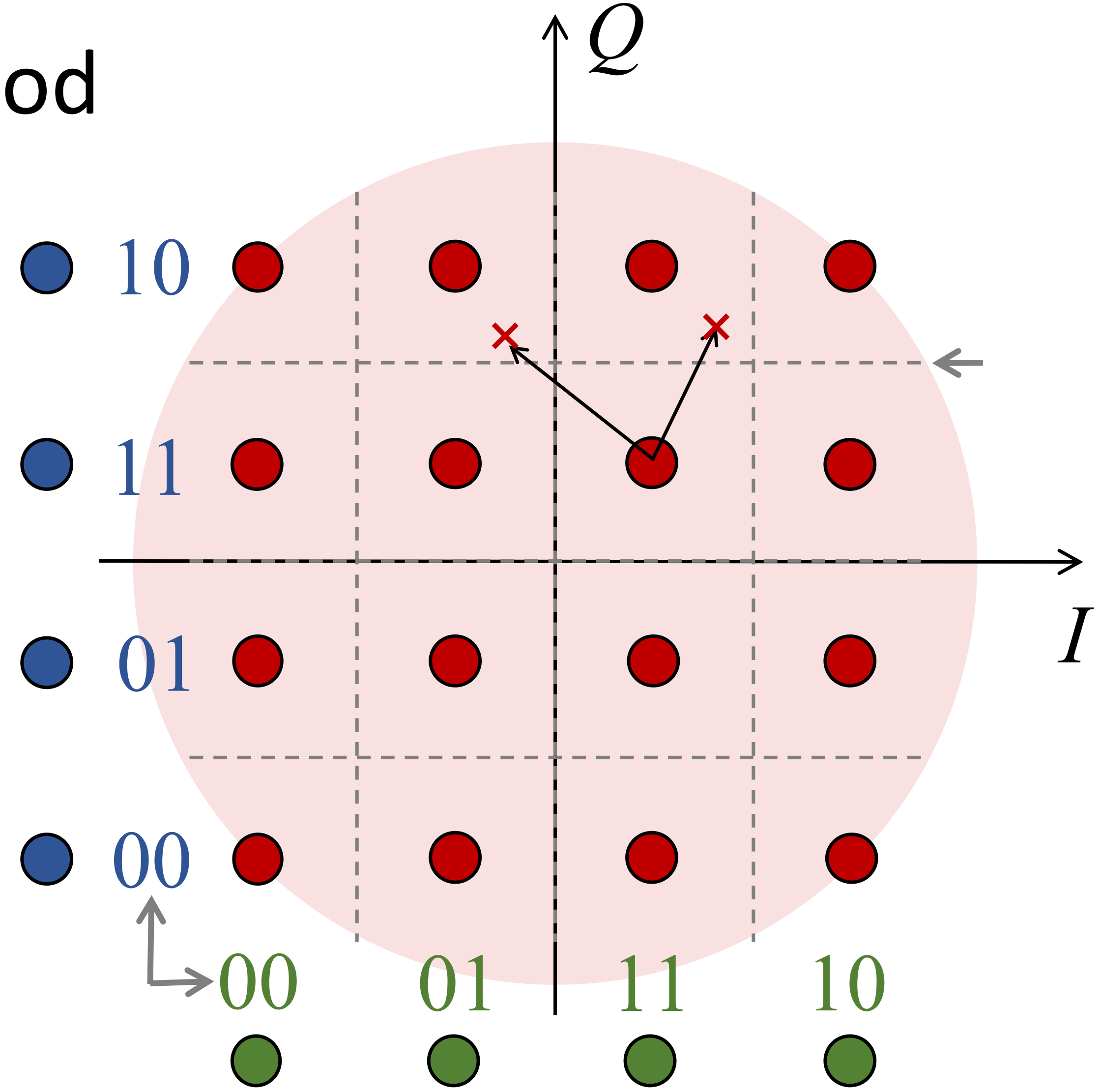
(Error Count)

$$BER = \frac{\#Errors}{\#Bits}$$

$$BER_{ref} = 10^{-3} \rightarrow 10^{-9}$$

FEC codes

$$\underbrace{P(E)_{bit}}_{BER} \approx P(E)_{symbol}$$



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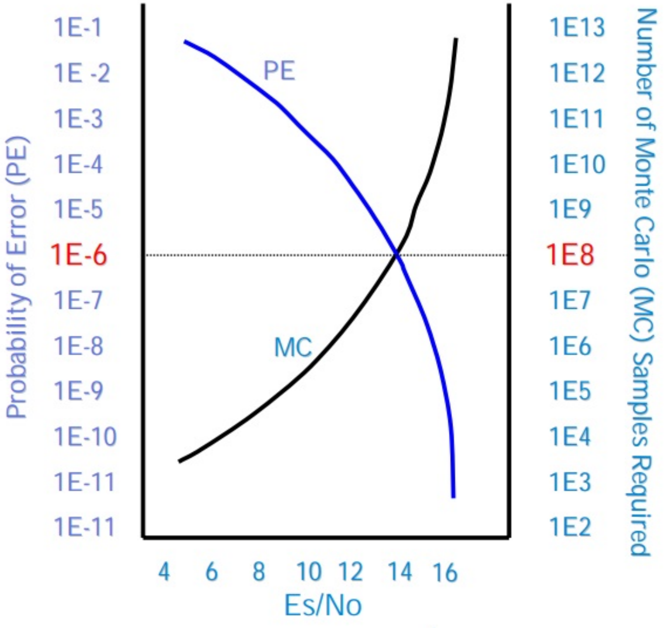
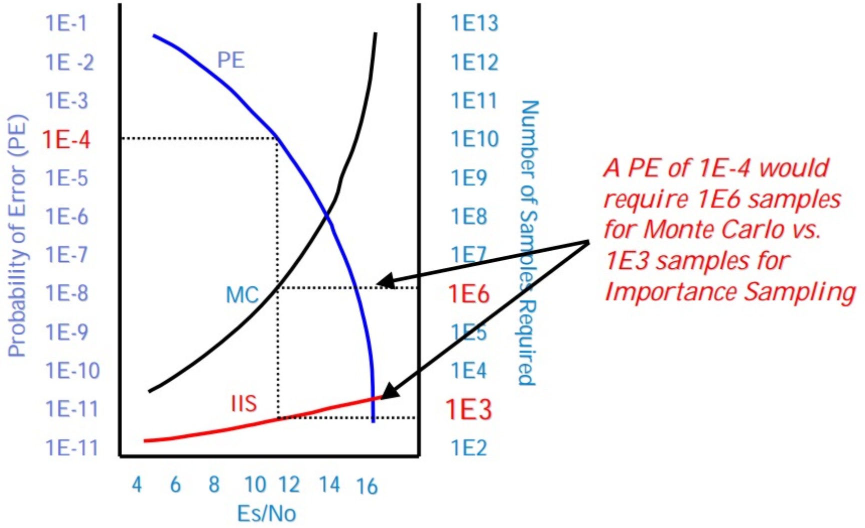
Number of Required Samples

Using Monte Carlo BER Estimation (99% Confidence)



BER - Using Importance Sampling

How to save simulation time…



Agilent EEsof EDA

Presentation on Error Rate Simulations

http://literature.cdn.keysight.com/litweb/pdf/5989-9111EN.pdf

References

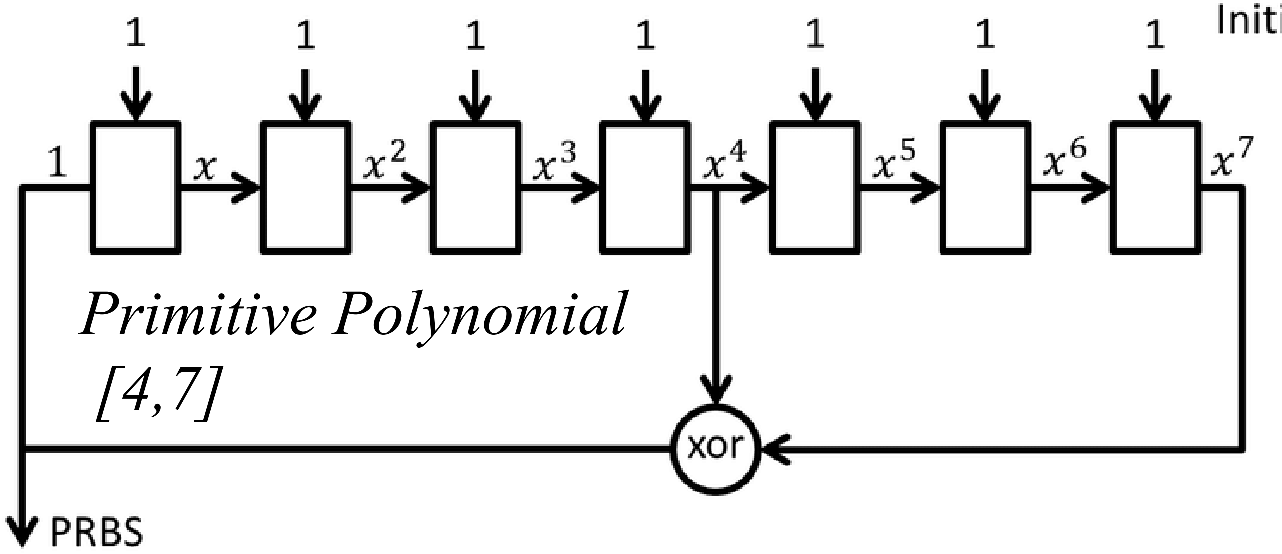
Dingqing Lu and Kung Yao, "Improved Importance Sampling Technique for Efficient Simulation of Digital Communication Systems", IEEE Journal on Selected Areas in Communications, Vol. 6, No. 1, pp. 67-75, January 1988

Dingqing Lu and Kung Yao, "Estimation Variance Bounds Of Importance Sampling Simulations in Digital Communcation Systems", IEEE Transactions on Communication, Vol. 39, No. 10, October, 1991

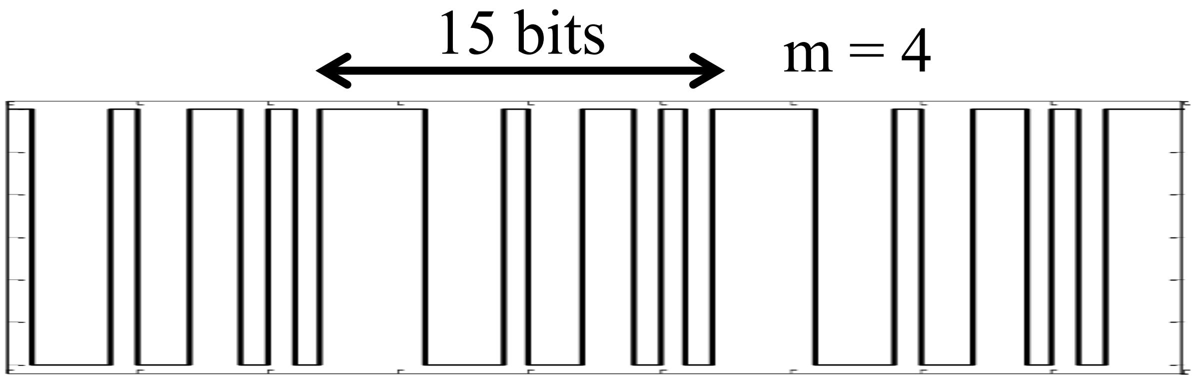
BIT ERROR RATIO (BER) ESTIMATION

Pseudo Random Bit Sequences (PRBS)

Initial State (Seed)



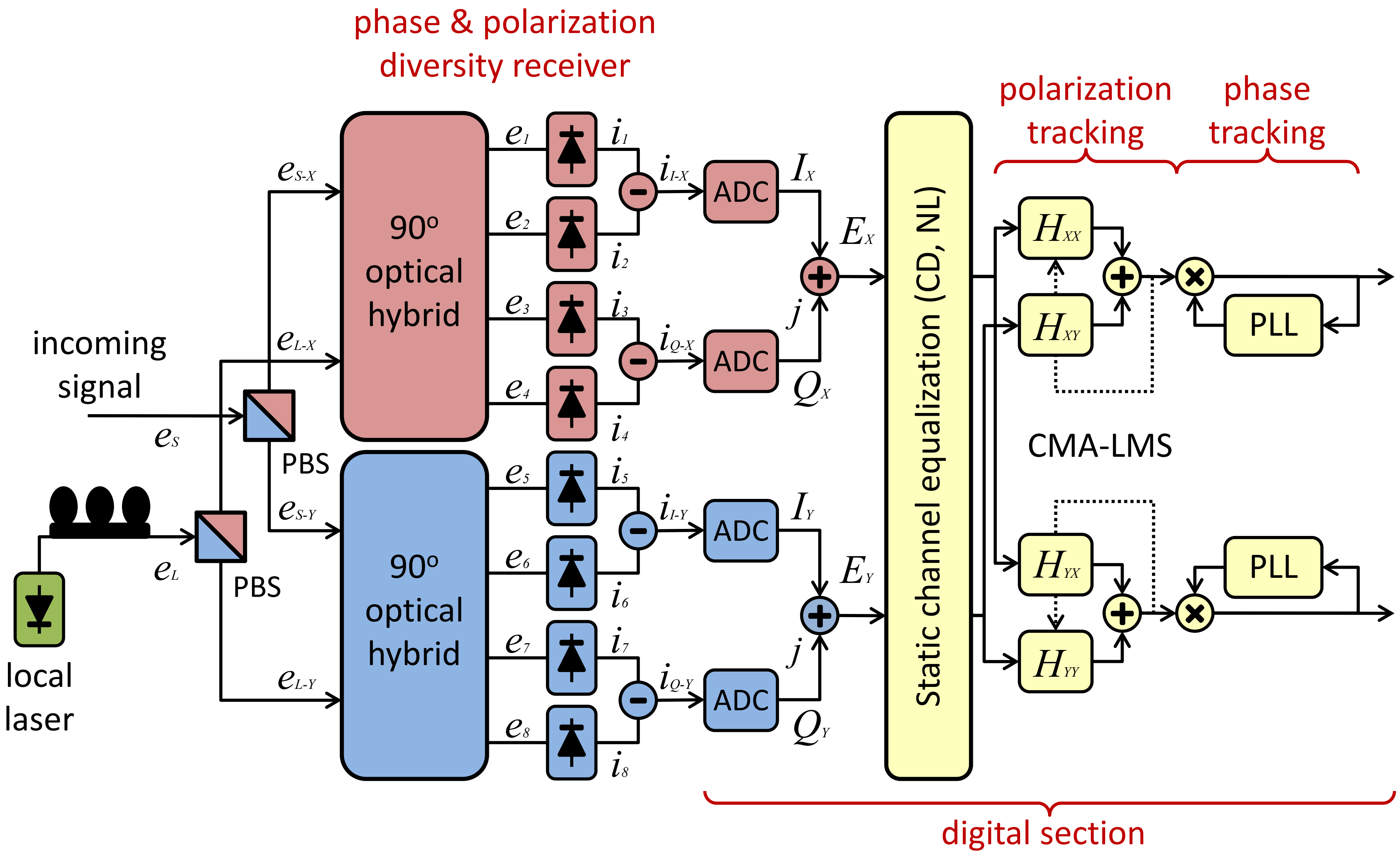
(Order)
m-stage feedback shift register



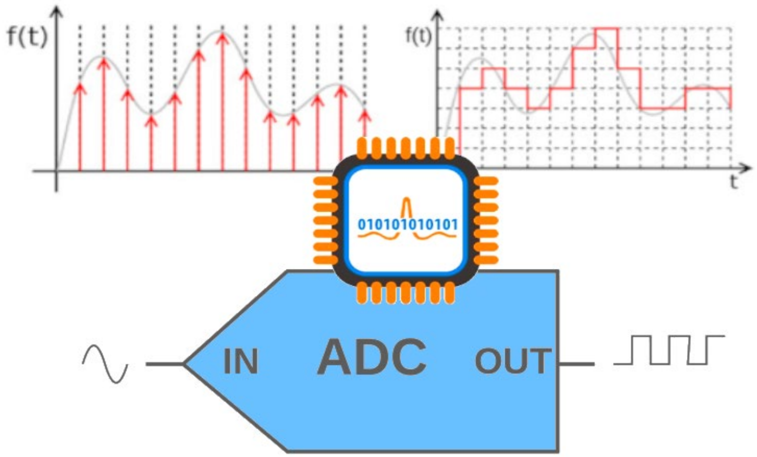
A maximal-length sequence generated will contain all but one *m*-bit combinations within a length of $\mathbf{2^m\text{-}1}$ **bits**.

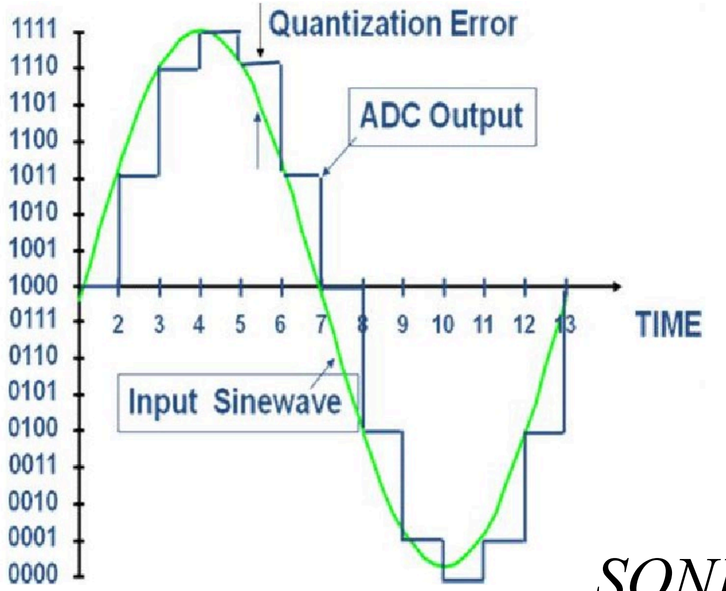
Michel C. Jeruchim et. al., "Simulation of Communication Systems", Kluwer Academic, Second Edition, 2002.

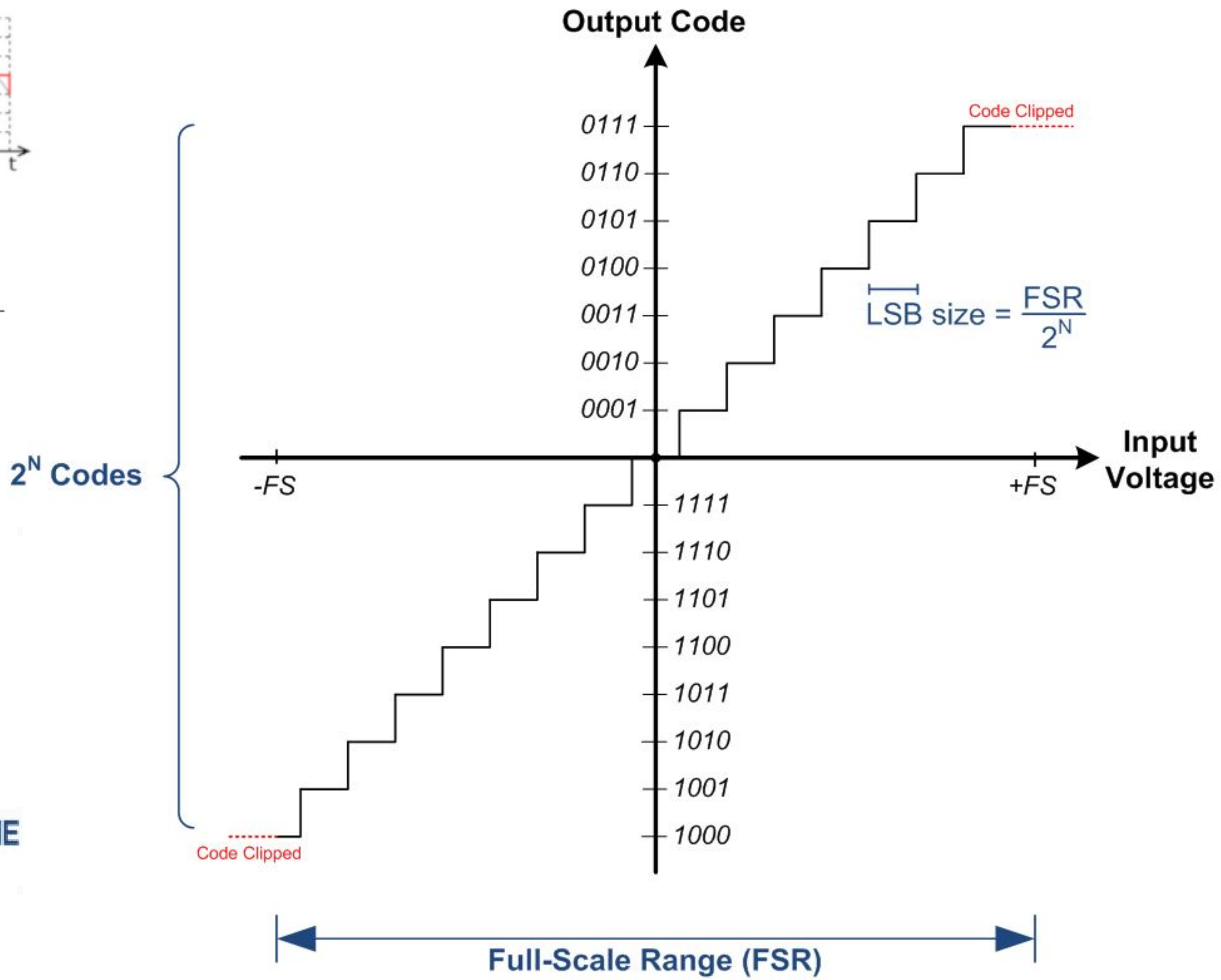
DIGITAL COHERENT RECEIVER



Analog to Digital Conversion







$$SQNR = 20\log\left(2^N\right) \rightarrow SQNR_{dB} \approx 6N$$

Chromatic Dispersion Compensation

Linear Regime

$$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} \xrightarrow{FT} \frac{\partial \mathcal{A}}{\partial z} = -j\omega^2\frac{\beta_2}{2}\mathcal{A}$$

$$\mathcal{A}(z,\omega)=\mathcal{A}(0,\omega)\underbrace{e^{-j\omega^2\frac{\beta_2}{2}z}}_{H(z,\omega)} \longrightarrow h(z,t)=\frac{1}{\left(j2\pi\beta_2 z\right)^{1/2}}e^{j\frac{t^2}{2\beta_2 z}}$$

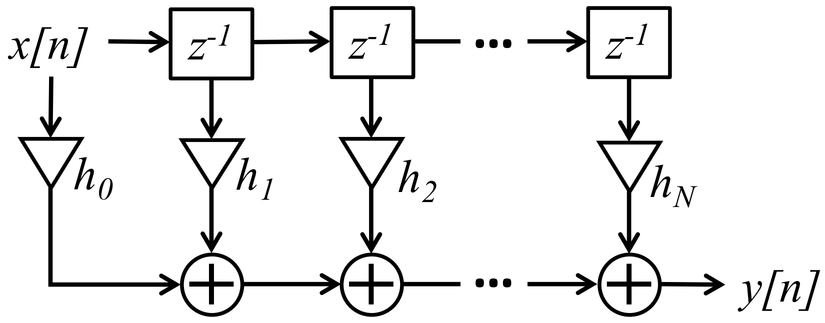
A signal sampled every T_{ADC} seconds can be recovered by applying a finite impulse response (FIR) filter to the signal with tap weights:

$$h[n]=\frac{1}{\sqrt{\rho}}e^{j\frac{\pi}{\rho}\left[n-\frac{N-1}{2}\right]^2}$$

$$n\in[0,N-1]$$

$$N=\lfloor|\rho|\rfloor$$

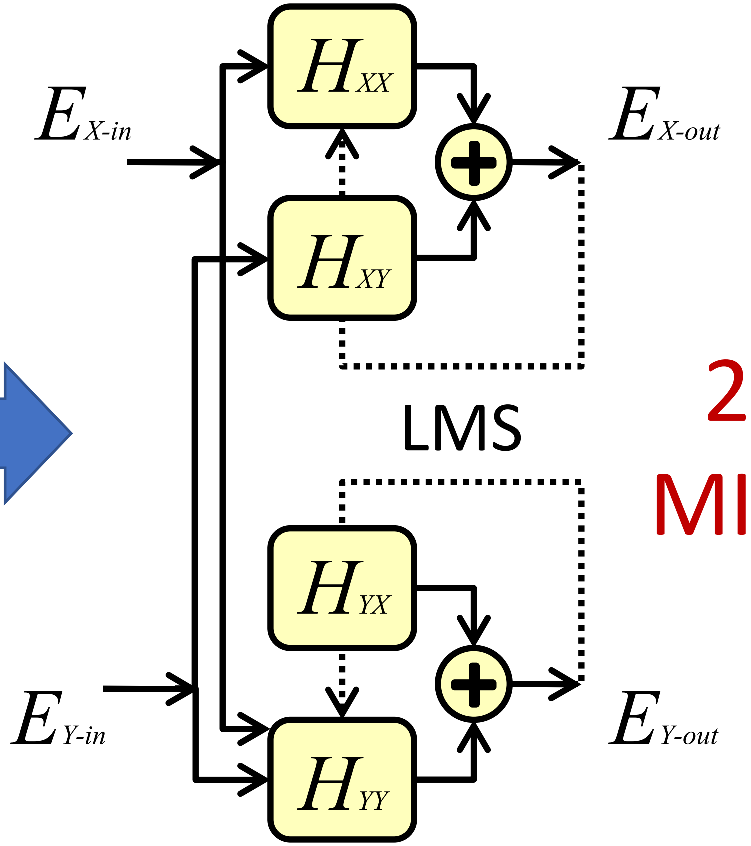
$$\rho=2\pi\beta_2\frac{L}{T_{ADC}^2}$$



N: Number of Taps

Polarization Tracking & PMD Compensation

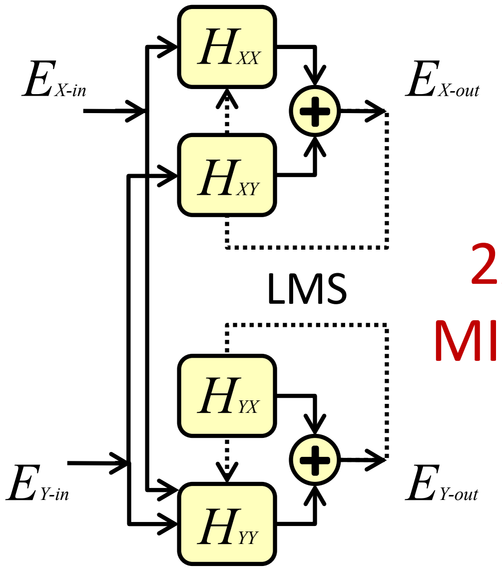
$$\begin{pmatrix} \sqrt{\alpha}\cdot e^{-j\Delta\theta(t)} & \sqrt{1-\alpha} \\ \sqrt{1-\alpha} & -\sqrt{\alpha}\cdot e^{j\Delta\theta(t)} \end{pmatrix}$$



FIR filters

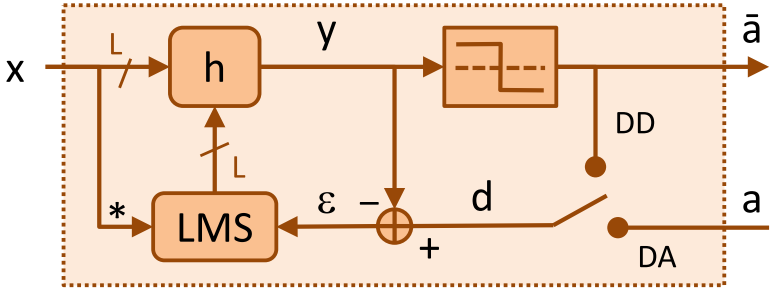
2x2 MIMO

Least Mean Square (LMS) Algorithm



FIR filters

2x2 MIMO



- Fast Convergence
- Training sequence required
- High computational cost
- Hingh sensitivity to frequency and phase misalignment (within PLL)

error function

$$\varepsilon_X = d - A_{X-out}$$
$$\varepsilon_Y = d - A_{Y-out}$$

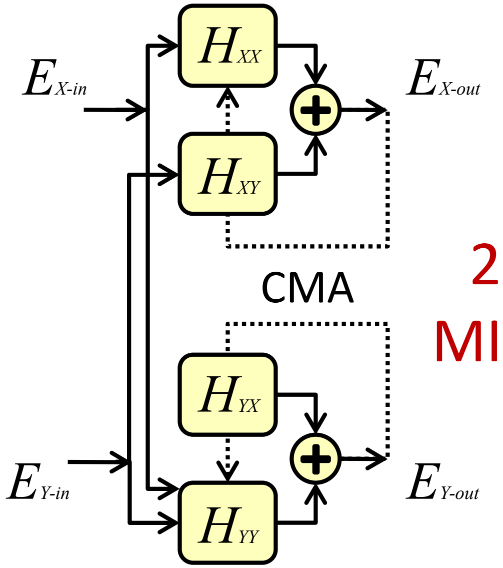
d: decided data

filter updating mechanism

$$h_{XX}^{k+1} \mapsto h_{XX}^{k} + \mu \cdot \varepsilon_X^k \cdot A_{X-in}^{*\,k}$$
$$h_{XY}^{k+1} \mapsto h_{XY}^{k} + \mu \cdot \varepsilon_X^k \cdot A_{Y-in}^{*\,k}$$
$$h_{YX}^{k+1} \mapsto h_{YX}^{k} + \mu \cdot \varepsilon_Y^k \cdot A_{X-in}^{*\,k}$$
$$h_{YY}^{k+1} \mapsto h_{YY}^{k} + \mu \cdot \varepsilon_Y^k \cdot A_{Y-in}^{*\,k}$$

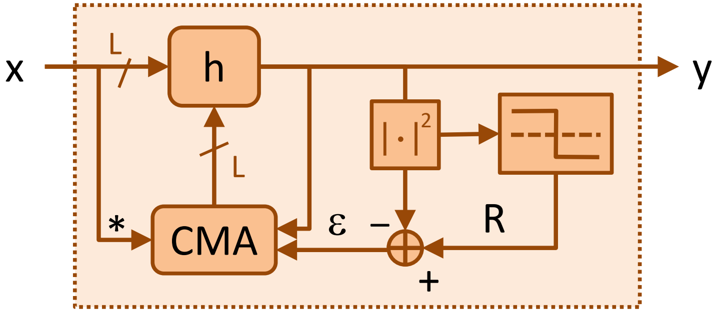
μ: Convergence parameter (~ 10^{-4})

Constant Modulus Algorithm (CMA)



FIR filters

2x2 MIMO



- Blind filter adaptation (no training sequence)
- Robust adaptative algorithm
- Independent of carrier frequency and phase (before PLL)
- Pre-convergence for QAM

error function

$$\varepsilon_X = \left(R^2 - \left|A_{X-out}\right|^2\right)$$

$$\varepsilon_Y = \left(R^2 - \left|A_{Y-out}\right|^2\right)$$

R: circle radius

filter updating mechanism

$$h_{XX}^{k+1} \mapsto h_{XX}^{k} + \mu \cdot \varepsilon_X^k \cdot A_{X-in}^{*\,k} A_{X-out}^{k}$$

$$h_{XY}^{k+1} \mapsto h_{XY}^{k} + \mu \cdot \varepsilon_X^k \cdot A_{Y-in}^{*\,k} A_{X-out}^{k}$$

$$h_{YX}^{k+1} \mapsto h_{YX}^{k} + \mu \cdot \varepsilon_Y^k \cdot A_{X-in}^{*\,k} A_{Y-out}^{k}$$

$$h_{YY}^{k+1} \mapsto h_{YY}^{k} + \mu \cdot \varepsilon_Y^k \cdot A_{Y-in}^{*\,k} A_{Y-out}^{k}$$

μ: Convergence parameter (~ 10^{-2})

Constant Modulus Algorithm (CMA)

$$E_{\parallel}=\frac{R}{\sqrt{2}}\left|A_S^{\parallel}(t)\right|\left|A_L\right|\begin{pmatrix}\sqrt{\alpha}\cdot e^{j\Delta\theta(t)}\\ \sqrt{1-\alpha}\end{pmatrix}e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\parallel}(t)-\theta_L(t)}^{\theta_{\parallel}(t)}\right)}$$

$$E_{\perp}=\frac{R}{\sqrt{2}}\left|A_S^{\perp}(t)\right|\left|A_L\right|\begin{pmatrix}\sqrt{1-\alpha}\\ -\sqrt{\alpha}\cdot e^{-j\Delta\theta(t)}\end{pmatrix}e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\perp}(t)-\theta_L(t)}^{\theta_{\perp}(t)}\right)}$$

$$\left|E_X^{\parallel}+E_X^{\perp}\right|^2=\left|R\sqrt{\frac{\alpha}{2}}\left|A_S^{\parallel}(t)\right|\left|A_L\right|e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\parallel}(t)-\theta_L(t)}^{\theta_{\parallel}(t)}+\Delta\theta(t)\right)}+R\sqrt{\frac{1-\alpha}{2}}\left|A_S^{\perp}(t)\right|\left|A_L\right|e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\perp}(t)-\theta_L(t)}^{\theta_{\perp}(t)}\right)}\right|^2=$$

$$=\frac{R^2}{2}\left(\alpha\left|A_S^{\parallel}(t)\right|^2+(1-\alpha)\left|A_S^{\perp}(t)\right|^2\right)\left|A_L\right|^2+R^2\sqrt{\alpha}\sqrt{1-\alpha}\left|A_S^{\parallel}(t)\right|\left|A_S^{\perp}(t)\right|\left|A_L\right|^2\cos\left(\theta_S^{\parallel}(t)-\theta_S^{\perp}(t)+\Delta\theta(t)\right)$$

$$\left|E_Y^{\parallel}+E_Y^{\perp}\right|^2=\left|R\sqrt{\frac{1-\alpha}{2}}\left|A_S^{\parallel}(t)\right|\left|A_L\right|e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\parallel}(t)-\theta_L(t)}^{\theta_{\parallel}(t)}\right)}-R\sqrt{\frac{\alpha}{2}}\left|A_S^{\perp}(t)\right|\left|A_L\right|e^{j\left(\overbrace{\omega_{FI}t+\theta_S^{\perp}(t)-\theta_L(t)}^{\theta_{\perp}(t)}-\Delta\theta(t)\right)}\right|^2=$$

$$=\frac{R^2}{2}\left((1-\alpha)\left|A_S^{\parallel}(t)\right|^2+\alpha\left|A_S^{\perp}(t)\right|^2\right)\left|A_L\right|^2-R^2\sqrt{\alpha}\sqrt{1-\alpha}\left|A_S^{\parallel}(t)\right|\left|A_S^{\perp}(t)\right|\left|A_L\right|^2\cos\left(\theta_S^{\parallel}(t)-\theta_S^{\perp}(t)+\Delta\theta(t)\right)$$

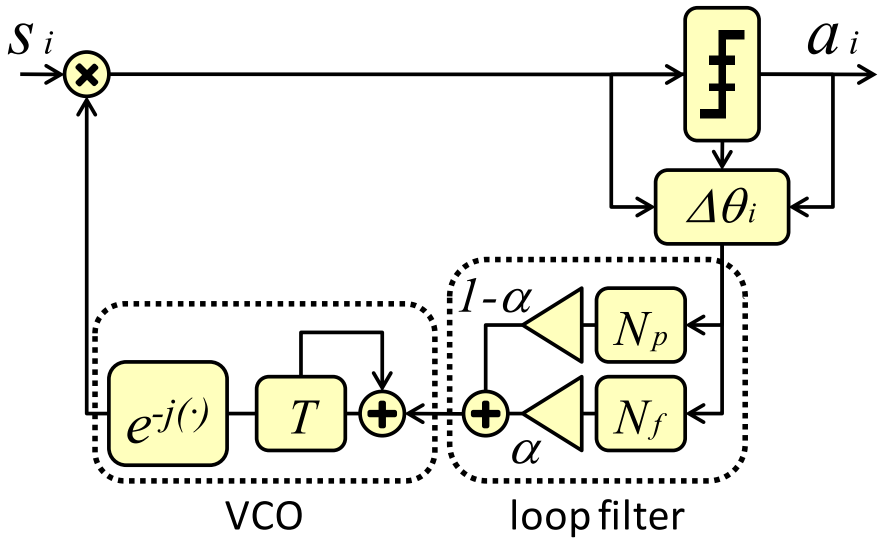
$$\left|A_S^{\parallel}(t)\right|^2=\left|A_S^{\perp}(t)\right|^2$$ ← **QPSK**

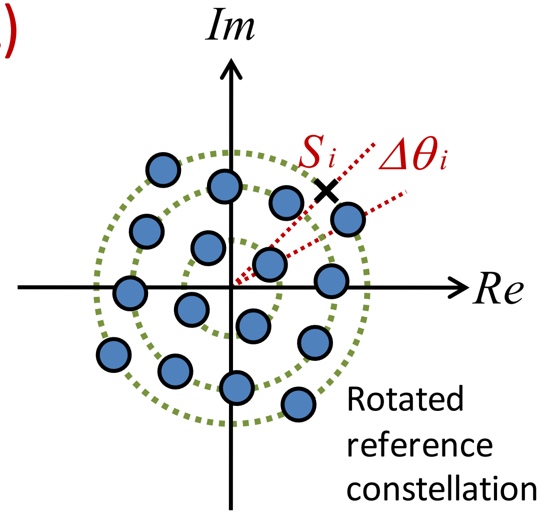
$$\left|E_X^{\parallel}+E_X^{\perp}\right|^2=\frac{R^2}{2}\left|A_S(t)\right|^2\left|A_L\right|^2+R^2\sqrt{\alpha}\sqrt{1-\alpha}\left|A_S(t)\right|^2\left|A_L\right|^2\cos\left(\theta_S^{\parallel}(t)-\theta_S^{\perp}(t)+\Delta\theta(t)\right)$$

$$\left|E_Y^{\parallel}+E_Y^{\perp}\right|^2=\frac{R^2}{2}\left|A_S(t)\right|^2\left|A_L\right|^2-R^2\sqrt{\alpha}\sqrt{1-\alpha}\left|A_S(t)\right|^2\left|A_L\right|^2\cos\left(\theta_S^{\parallel}(t)-\theta_S^{\perp}(t)+\Delta\theta(t)\right)$$

Frequency & Phase Estimation

Decision-Directed Phase-Locked Loop (DD-PLL)





When LMS is used, PLL is placed inside the LMS loop increasing the system's instability. CMA avoids this situation.

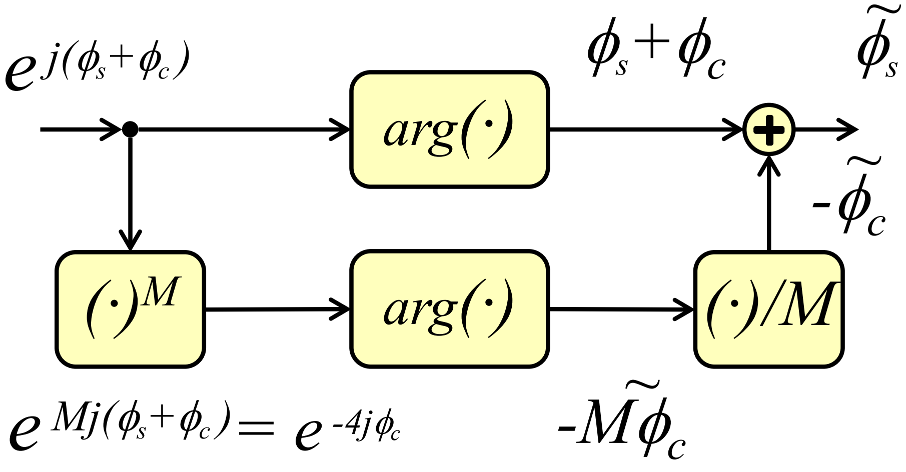
$\alpha \sim 0.95$
N_f ~ 1000 samples
N_p ~ 10 samples

Reference constellation steps ($\pi/20$) following the smallest mean-square error (Maximum-Likekihood)

- Constellation spinning at IF
- Constellation wiggling due to phase noise
- Constellation rotated due to phase uncertainity
- QAM grid decision
- Gray code
- Short training sequence

Viterbi & Viterbi Algorithm

Viterbi & Viterbi Algorithm

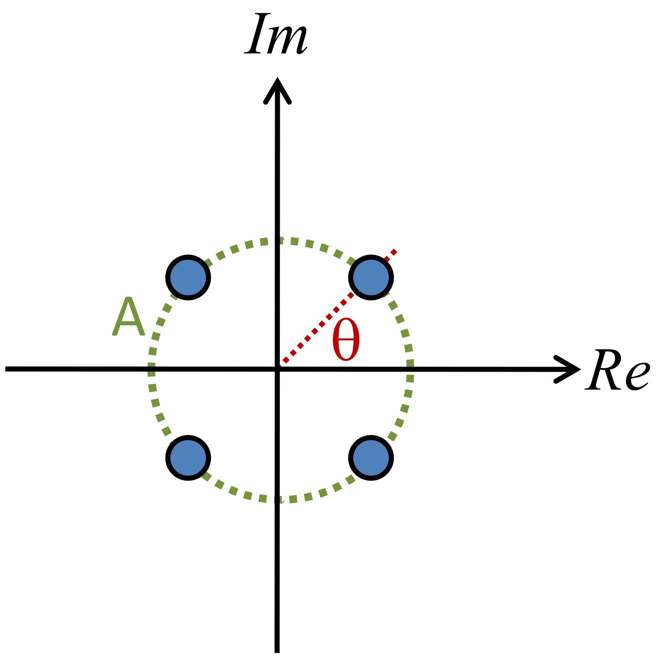


PSK

$$\phi_S = \frac{\pi}{M}(2m+1) \qquad , m = \{0 \ldots M\}$$

- Very symple implementation
- Works only for QPSK
- Multi-modulus variations for QAM

QPSK



$$\phi_S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$